

# CALCULUS INTEGRALS

## DEFINITE INTEGRAL DEFINITION

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \Delta x$$

where  $\Delta x = \frac{b-a}{n}$  and  $x_k = a + k\Delta x$

## FUNDAMENTAL THEOREM OF CALCULUS

$$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a)$$

where  $f$  is continuous on  $[a, b]$  and  $F' = f$

## INTEGRATION PROPERTIES

$$\int_a^b cf(x) dx = c \int_a^b f(x) dx$$

$$\int_a^b f(x) \pm g(x) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$$

$$\int_a^a f(x) dx = 0 \text{ and } \int_a^b f(x) dx = -\int_b^a f(x) dx$$

$$\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$$

## COMMON INTEGRALS

$$\int k dx = kx + C$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C, n \neq -1$$

$$\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|ax+b| + C$$

$$\int \ln(x) dx = x \ln(x) - x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \tan x dx = \ln|\sec x| + C$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C$$

$$\int \frac{1}{a^2+u^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}} dx = \sin^{-1}\left(\frac{u}{a}\right) + C$$

## APPROXIMATING DEFINITE INTEGRALS

Left-hand and right-hand rectangle approximations

$$L_n = \Delta x \sum_{k=0}^{n-1} f(x_k) \quad R_n = \Delta x \sum_{k=1}^n f(x_k)$$

Midpoint Rule

$$M_n = \Delta x \sum_{k=0}^{n-1} f\left(\frac{x_k + x_{k+1}}{2}\right)$$

Trapezoid Rule

$$T_n = \frac{\Delta x}{2} (f(x_0) + 2f(x_1) + 2f(x_2) + \dots + f(x_n))$$

## TRIGONOMETRIC SUBSTITUTION

EXPRESSION	SUBSTITUTION	EXPRESSION EVALUATION	IDENTITY USED
$\sqrt{a^2 - x^2}$	$x = a \sin \theta$ $dx = a \cos \theta d\theta$	$\sqrt{a^2 - a^2 \sin^2 \theta}$ $= a \cos \theta$	$1 - \sin^2 \theta$ $= \cos^2 \theta$
$\sqrt{x^2 - a^2}$	$x = a \sec \theta$ $dx = a \sec \theta \tan \theta d\theta$	$\sqrt{a^2 \sec^2 \theta - a^2}$ $= a \tan \theta$	$\sec^2 \theta - 1$ $= \tan^2 \theta$
$\sqrt{a^2 + x^2}$	$x = a \tan \theta$ $dx = a \sec^2 \theta d\theta$	$\sqrt{a^2 + a^2 \tan^2 \theta}$ $= a \sec \theta$	$1 + \tan^2 \theta$ $= \sec^2 \theta$

## APPROXIMATION BY SIMPSON RULE FOR EVEN N

$$S_n = \frac{\Delta x}{3} (f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n))$$

## INTEGRATION BY SUBSTITUTION

$$\int_a^b f(g(x)) g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$$

where  $u = g(x)$  and  $du = g'(x) dx$

## INTEGRATION BY PARTS

$$\int u dv = uv - \int v du \quad \text{where } v = \int dv$$

or

$$\int f(x) g'(x) dx = f(x) g(x) - \int f'(x) g(x) dx$$

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## DERIVATIVE DEFINITION

$$\frac{d}{dx}(f(x)) = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

## BASIC PROPERTIES

$$(cf(x))' = c(f'(x))$$

$$(f(x) \pm g(x))' = f'(x) \pm g'(x)$$

$$\frac{d}{dx}(c) = 0$$

## MEAN VALUE THEOREM

If  $f$  is differentiable on the interval  $(a, b)$  and continuous at the end points there exists a  $c$  in  $(a, b)$  such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

## PRODUCT RULE

$$(f(x)g(x))' = f(x)'g(x) + f(x)g(x)'$$

## QUOTIENT RULE

$$\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

## POWER RULE

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

## CHAIN RULE

$$\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$$

## LIMIT EVALUATION METHOD – FACTOR AND CANCEL

$$\lim_{x \rightarrow -3} \frac{x^2 - x - 12}{x^2 + 3x} = \lim_{x \rightarrow -3} \frac{(x+3)(x-4)}{x(x+3)} = \lim_{x \rightarrow -3} \frac{(x-4)}{x} = \frac{7}{3}$$

## L'HOPITAL'S RULE

$$\text{If } \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0} \text{ or } \frac{\pm\infty}{\pm\infty} \text{ then } \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

## COMMON DERIVATIVES

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(a^x) = a^x \ln(a)$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}, x > 0$$

$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$$

$$\frac{d}{dx}(\log_a(x)) = \frac{1}{x \ln(a)}$$

## CHAIN RULE AND OTHER EXAMPLES

$$\frac{d}{dx}([f(x)]^n) = n[f(x)]^{n-1}f'(x)$$

$$\frac{d}{dx}(e^{f(x)}) = f'(x)e^{f(x)}$$

$$\frac{d}{dx}(\ln[f(x)]) = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}(\sin[f(x)]) = f'(x)\cos[f(x)]$$

$$\frac{d}{dx}(\cos[f(x)]) = -f'(x)\sin[f(x)]$$

$$\frac{d}{dx}(\tan[f(x)]) = f'(x)\sec^2[f(x)]$$

$$\frac{d}{dx}(\sec[f(x)]) = f'(x)\sec[f(x)]\tan[f(x)]$$

$$\frac{d}{dx}(\tan^{-1}[f(x)]) = \frac{f'(x)}{1+[f(x)]^2}$$

$$\frac{d}{dx}(f(x)^{g(x)}) = f(x)^{g(x)} \left( \frac{g(x)f'(x)}{f(x)} + \ln(f(x))g'(x) \right)$$

## PROPERTIES OF LIMITS

These properties require that the limit of  $f(x)$  and  $g(x)$  exist

$$\lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} f(x)$$

$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} [f(x)g(x)] = \lim_{x \rightarrow a} f(x) \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} \left[ \frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \text{ if } \lim_{x \rightarrow a} g(x) \neq 0$$

$$\lim_{x \rightarrow a} [f(x)]^n = \left[ \lim_{x \rightarrow a} f(x) \right]^n$$

## LIMIT EVALUATIONS AT $+\infty$

$$\lim_{x \rightarrow \infty} e^x = \infty \text{ and } \lim_{x \rightarrow -\infty} e^x = 0$$

$$\lim_{x \rightarrow \infty} \ln(x) = \infty \text{ and } \lim_{x \rightarrow 0^+} \ln(x) = -\infty$$

$$\text{If } r > 0 \text{ then } \lim_{x \rightarrow \infty} \frac{c}{x^r} = 0$$

$$\text{If } r > 0 \text{ \& } x^r \text{ is real for } x < 0 \text{ then } \lim_{x \rightarrow -\infty} \frac{c}{x^r} = 0$$

$$\lim_{x \rightarrow \pm\infty} x^r = \infty \text{ for even } r$$

$$\lim_{x \rightarrow \infty} x^r = \infty \text{ \& } \lim_{x \rightarrow -\infty} x^r = -\infty \text{ for odd } r$$

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